CALCULUS MEMO

May/June 2019

QUESTION/*VRAAG* 7

7.1	$f(x)=x^2+2$ $f(x+h)=(x+h)^2+2$ $=x^2+2xh+h^2+2$ $f(x+h)-f(x)=x^2+2xh+h^2+2-(x^2+2)$ $=2xh+h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2}{h}$ $=\lim_{h\to 0}\frac{h(2x+h)}{h}$ $=\lim_{h\to 0}(2x+h)$ $=2x$	✓ $x^2+2xh+h^2+2$ ✓ $\lim_{h\to 0}\frac{2xh+h^2}{h}$ ✓ $\lim_{h\to 0}\frac{h(2x+h)}{h}$ ✓ answer (4)
	OR/*OF* $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2+2-(x^2+2)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2}{h}$ $=\lim_{h\to 0}\frac{h(2x+h)}{h}$ $=\lim_{h\to 0}(2x+h)$ $=2x$	**OR/*OF*** ✓ $x^2+2xh+h^2+2$ ✓ $\lim_{h\to 0}\frac{2xh+h^2}{h}$ ✓ $\lim_{h\to 0}\frac{h(2x+h)}{h}$ ✓ answer (4)

7.2.1	$y=4x^3+2x^{-1}$ $\frac{dy}{dx}=12x^2-2x^{-2}$	✓ $+2x^{-1}$ ✓ $12x^2$ ✓ $-2x^{-2}$ (3)
7.2.2	$y=4\sqrt[3]{x}+(3x^3)^2$ $=4x^{\frac{1}{3}}+9x^6$ $\frac{dy}{dx}=\frac{4}{3}x^{-\frac{2}{3}}+54x^5$	✓ $4x^{\frac{1}{3}}$ ✓ $9x^6$ ✓ $\frac{4}{3}x^{-\frac{2}{3}}$ ✓ $54x^5$ (4)
7.3	Point of contact: (1 ; 5) $m=2$ $y-y_1=m(x-x_1)$ or $y=2x+c$ $y-5=2(x-1)$ $\quad 5=2+c$ $c=3$ $y=2x+3$ $\quad y=2x+3$	✓ $m=2$ ✓ substitution of (1 ; 5) ✓ answer (3) **[14]**

NSC JUNE 2021

QUESTION/*VRAAG* 8

8.1	$f(x)=3x^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{3x^2+6xh+3h^2-3x^2}{h}$ $=\lim_{h\to 0}\frac{6xh+3h^2}{h}$ $=\lim_{h\to 0}\frac{h(6x+3h)}{h}$ $=6x$	✓ substitution ✓ expansion ✓ simplification ✓ $\lim_{h\to 0}\frac{h(6x+3h)}{h}$ ✓$6x$ (5)
8.2.1	$f(x)=x^2-3+9x^{-2}$ $f'(x)=2x-18x^{-3}$	✓ $9x^{-2}$ ✓$2x$ ✓ $-18x^{-3}$ (3)

8.2.2	$g(x)=(\sqrt{x}+3)(\sqrt{x}-1)$ $g(x)=x+2x^{\frac{1}{2}}-3$ $g'(x)=1+x^{-\frac{1}{2}}$	✓ x ✓ $2x^{\frac{1}{2}}$ ✓1 ✓ $x^{-\frac{1}{2}}$ (4)
		[12]

NSC NOV 2020

QUESTION/*VRAAG* 7

Penalty of – 1 for notation only in 7.1

7.1	$f(x)=2x^2-1$ $f(x+h)=2(x+h)^2-1$ $=2(x^2+2xh+h^2)^2-1$ $=2x^2+4xh+2h^2-1$ $f(x+h)-f(x)=2x^2+4xh+2h^2-1-(2x^2-1)$ $=2x^2+4xh+2h^2-1-2x^2+1$ $=4xh+2h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h)}{h}$ $=\lim_{h\to 0}(4x+2h)$ $=4x$	✓ $2x^2+4xh+2h^2-1$ ✓ $4xh+2h^2$ ✓ substitution ✓ simplification ✓ answer (5)
7.2.1	$\frac{d}{dx}\left(\sqrt[5]{x^2}+x^3\right)$ $=\frac{d}{dx}\left(x^{\frac{2}{5}}+x^3\right)$ $\frac{dy}{dx}=\frac{2}{5}x^{-\frac{3}{5}}+3x^2$	✓ $x^{\frac{2}{5}}$ ✓ $\frac{2}{5}x^{-\frac{3}{5}}$ ✓ $3x^2$ (3)
7.2.2	$f(x)=\frac{4x^2-9}{4x+6}$ $=\frac{(2x-3)(2x+3)}{2(2x+3)}$ $=\frac{2x-3}{2}$ $=x-\frac{3}{2}$ $f'(x)=1$	✓ $(2x-3)(2x+3)$ ✓ $2(2x+3)$ ✓ simplification to two separate terms ✓ answer (4)
		[12]

QUESTION/VRAAG 9

9.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{2x^2+4xh+2h^2-3x-3h-2x^2+3x}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h-3)}{h}$ $=\lim_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$ **OR/OF** $f(x)=2x^2-3x$ $f(x+h)=2(x+h)^2-3(x+h)$ $f(x+h)=2x^2+4xh+2h^2-3x-3h$ $f(x+h)-f(x)=4xh+2h^2-3h$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h-3)}{h}$ $=\lim_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$	✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5) **OR/OF** ✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5)
9.2.1	$y=4x^5-6x^4+3x$ $\frac{dy}{dx}=20x^4-24x^3+3$	✓ $20x^4$ ✓ $-24x^3$ ✓ 3 (3)

9.2.2	$D_x\left[\frac{-\sqrt[3]{x}}{2}+\left(\frac{1}{3x}\right)^2\right]$ $D_x\left[\frac{-x^{\frac{1}{3}}}{2}+\frac{x^{-2}}{9}\right]$ $D_x\left[-\frac{1}{2}x^{\frac{1}{3}}+\frac{1}{9}x^{-2}\right]$ $=-\frac{1}{6}x^{-\frac{2}{3}}-\frac{2x^{-3}}{9}$ $=-\frac{1}{6x^{\frac{2}{3}}}-\frac{2}{9x^3}$	✓ $\frac{-x^{\frac{1}{3}}}{2}$ ✓ $\frac{x^{-2}}{9}$ ✓ $-\frac{1}{6}x^{-\frac{2}{3}}$ ✓ $-\frac{2x^{-3}}{9}$ (4)
		[12]

MAY /JUNE 2019

QUESTION/*VRAAG* 8

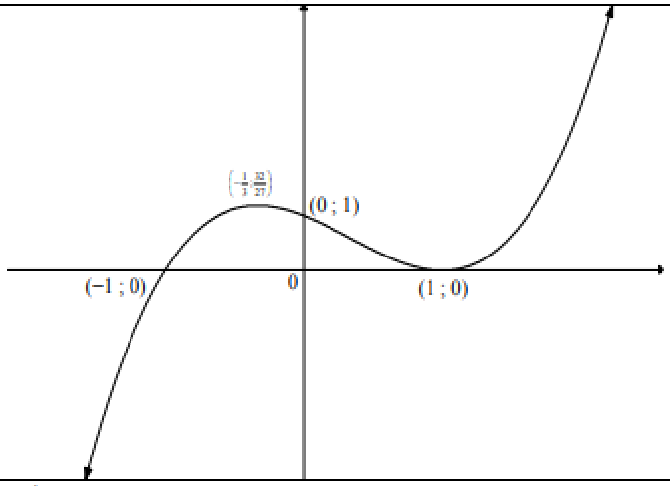
8.1	$h(x)=-2(x+\frac{3}{2})(x-1)(x+3)$ $h(x)=-(2x+3)(x^2+2x-3)$ $h(x)=-2x^3-7x^2+9$ **OR/*OF*** $h(x)=-(2x+3)(x-1)(x+3)$ $h(x)=-(2x+3)(x^2+2x-3)$ $h(x)=-2x^3-7x^2+9$	✓✓ $-2(x+\frac{3}{2})(x-1)(x+3)$ ✓correct simplification (3) **OR/*OF*** ✓✓ $-(2x+3)(x-1)(x+3)$ ✓ correct simplification (3)
8.2	$h'(x)=-6x^2-14x$ $-6x^2-14x=0$ $-2x(3x+7)=0$ $x=0$ or $x=-\frac{7}{3}$	✓ first derivative ✓ $=0$ ✓ both answers (3)
8.3	$x<-\frac{7}{3}$ or $x>0$ **OR/*OF*** $x\in\left(-\infty\,;-\frac{7}{3}\right)\cup(0\,;\infty)$	✓✓ answer (2) **OR/*OF*** ✓✓ answer (2)

8.4	$y = 4x + 7$ $-6x^2 - 14x = 4$ $0 = 6x^2 + 14x + 4$ $0 = 3x^2 + 7x + 2$ $0 = (3x+1)(x+2)$ $x = -\frac{1}{3}$ or $x = -2$	✓ $y = 4x + 7$ ✓ $h'(x) = 4$ ✓ standard form ✓ both answers (4) **[12]**

JUNE 2017

QUESTION/*VRAAG* 9

9.1	(0 ; 1)	✓ answer (1)
9.2	$f(x) = x^3 - x^2 - x + 1$ $f(x) = x^2(x-1) - (x-1)$ $f(x) = (x-1)(x^2-1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0) **OR** $f(x) = x^3 - x^2 - x + 1$ $f(x) = (x-1)(x^2-1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0) **OR**	✓ $(x-1)$ ✓ (x^2-1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5) ✓ $(x-1)$ ✓ (x^2-1) ✓ $(x-1)(x-1)(x+1)$ ✓(–1; 0) ✓(1; 0) (5)

	$f(x) = x^3 - x^2 - x + 1$ $f(x) = (x+1)(x^2 - 2x + 1)$ $f(x) = (x+1)(x-1)(x-1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: $(-1;\,0)$; $(1;\,0)$	✓ $(x+1)$ ✓ $(x^2 - 2x + 1)$ ✓ $(x-1)(x-1)(x+1)$ ✓$(-1;\,0)$ ✓$(1;\,0)$ (5)
9.3	$f(x) = x^3 - x^2 - x + 1$ $f'(x) = 3x^2 - 2x - 1$ $f'(x) = 0$ $(3x+1)(x-1) = 0$ $x = -\frac{1}{3}$ or $x = 1$ $y = \frac{32}{27}$ $y = 0$ $\left(-\frac{1}{3};\frac{32}{27}\right)$ $(1;\,0)$	✓ $f'(x) = 3x^2 - 2x - 1$ ✓ $f'(x) = 0$ ✓factorisation ✓x value ✓x value ✓$y = \frac{32}{27}$ (6)
9.4		✓y– and x–intercepts ✓shape ✓turning points (3)
9.5	$f'(x) < 0$ $-\frac{1}{3} < x < 1$ **OR/*OF*** $(-\frac{1}{3}\,;\,1)$	✓ $x > -\frac{1}{3}$ ✓ $x < 1$ (2) ✓ $(-\frac{1}{3}$ ✓ $1)$ (2) **[17]**

NOV 2020

QUESTION/*VRAAG* 8

8.1	$-1 < x < 2$	✓✓ answer (2)
8.2	$x = \frac{-1+2}{2}$ $x = \frac{1}{2}$ **Answer Only: Full Marks**	✓ method ✓ answer (2)
8.3	From the graph $x > \frac{1}{2}$ **Answer Only: Full Marks**	✓✓ answer (2)
8.4	$g(x) = ax^3 + bx^2 + cx$ $g'(x) = 3ax^2 + 2bx + c = -6x^2 + 6x + 12$ $3a = -6$. $2b = 6$ $c = 12$ $a = -2$ $b = 3$ $g(x) = -2x^3 + 3x^2 + 12x$	✓ $g'(x) = 3ax^2 + 2bx + c$ ✓ $a = -2$ ✓ $b = 3$ ✓ $g(x) = -2x^3 + 3x^2 + 12x$ (4)
8.5	$g'\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) + 12$ $m = \frac{27}{2}$ or 13,5 $y = -2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 12\left(\frac{1}{2}\right)$ $y = \frac{13}{2}$ or 6,5 $y - y_1 = m(x - x_1)$ $y - 6{,}5 = 13{,}5(x - 0{,}5)$ $y = 13{,}5x - 0{,}25$	✓max gradient at $x = \frac{1}{2}$ ✓answer ✓ y value ✓substitution ✓answer (5)
		[15]

MAY/JUNE 2021

QUESTION/*VRAAG* 9

9.1	$f^{/}(x) = 6x^2 + 6x - 12$ $6x^2 + 6x - 12 = 0$ $x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $x = -2$ or $x = 1$ $y = 20$ or $y = -7$ $\therefore$ A(–2 ; 20) and B(1 ; –7)	✓ $6x^2 + 6x - 12$ ✓ $= 0$ ✓factors ✓ x -values ✓ y -values (5)
9.2	$f^{//}(x) = 12x + 6$ $12x + 6 > 0$ $12x > -6$ $x > -\frac{1}{2}$ **OR/OF** $x = \frac{-2+1}{2} = -\frac{1}{2}$ $\therefore x > -\frac{1}{2}$	✓$12x + 6$ ✓ $f^{//}(x) > 0$ ✓ $x > -\frac{1}{2}$ (3) **OR/OF** ✓ $x = -\frac{1}{2}$ ✓✓ $x > -\frac{1}{2}$ (3)
9.3	$f^{/}(2) = 24$ Equation of the tangent: $y - 4 = 24(x - 2)$ $y = 24x - 44$	✓ $f^{/}(2)$ ✓ 24 ✓ equation (3)
		[11]

NOV 2021

QUESTION/VRAAG 10

10.1	$h(x)=ax^3+bx^2$ $h^{/}(x)=3ax^2+2bx$ $h^{/}(4)=0$ $48a+8b=0$ $6a+b=0$ ...(1) $h(4)=32$ $64a+16b=32$ $4a+b=2$ ...(2) (1)– (2): $6a+b=0$ $4a+b=2$ $2a=-2$ $a=-1$ $4(-1)+b=2$ $b=6$	✓ $h^{/}(x)$ ✓ $h^{/}(4)=0$ ✓ $48a+8b=0$ or $6a+b=0$ ✓ $h(4)=32$ ✓ $64a+16b=32$ or $4a+b=2$ (5)
10.2	$h(x)=-x^3+6x^2$ $-x^3+6x^2=0$ $x^2(-x+6)=0$ $x=0$ or $x=6$ $\therefore$ A(6 ; 0)	✓ $-x^3+6x^2=0$ ✓ factors ✓ A(6 ; 0) (3)
10.3.1	$0<x<4$ or $0\le x\le 4$ **OR/OF** $x\in(0;4)$ or $x\in[0;4]$	✓critical values ✓notation (2) **OR/OF** ✓critical values ✓notation (2)
10.3.2	$x>2$ **OR/OF** $x\in(2;\infty)$	✓ 2 ✓ notation (2) **OR/OF** ✓ 2 ✓ notation (2)
10.4	$f(x)=h(x-1)=-(x-1)^3+6(x-1)^2$ $f(0)=7$ $7<k<32$ or $k\in(7;32)$	✓ $k<32$ ✓ new y-intercept = 7 ✓ $7<k<32$ (3)
		[15]

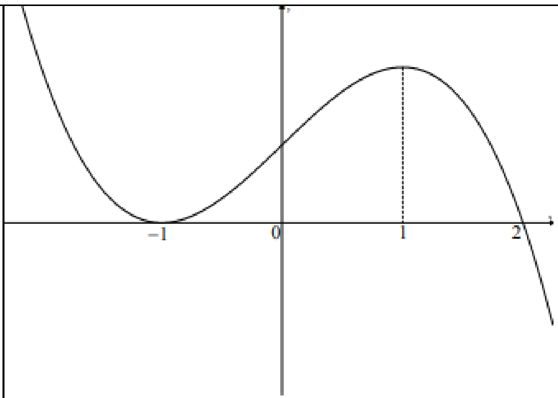
Nov 2018

QUESTION/*VRAAG* 9

9.1.1	$g(x)=(x+5)(x-x_1)^2$ $20=5(x_1)^2$ $x_1^2=4$ $x_1=2$ $g(x)=(x+5)(x-2)^2$ $g(x)=(x+5)(x^2-4x+4)$ $g(x)=x^3+x^2-16x+20$	✓ $(x+5)$ ✓repeated root ✓ $x_1=2$ ✓ $g(x)=(x+5)(x^2-4x+4)$ (4)
9.1.2	$g(x)=x^3+x^2-16x+20$ $g'(x)=3x^2+2x-16$ $3x^2+2x-16=0$ $(3x+8)(x-2)=0$ $x=\frac{-8}{3}$ or $x=2$ $R\left(\frac{-8}{3};\frac{1372}{27}\right)$ or $R(-2{,}67;50{,}81)$ $P(2;0)$	✓derivative ✓equating to zero ✓factors ✓co-ordinates of R ✓ co-ordinates of P (5)
9.1.3	$g''(x)=6x+2$ $g''(0)=2$ $\therefore$ concave up **OR/OF** $g''(x)=6x+2$ $6x+2=0$ $x=-\frac{1}{3}$ is the point of inflection $\therefore$ concave up	✓ $g''(x)=6x+2$ ✓ $g''(0)=2$ ✓conclusion (3) **OR/OF** ✓ $g''(x)=6x+2$ ✓ $x=-\frac{1}{3}$ ✓conclusion (3)

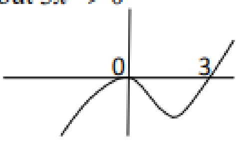
JUNE 2021

QUESTION/*VRAAG* 10

10.1		✓ $x=-1$ and $x=2$ ✓ TP at $x=-1$ ✓ TP at $x=1$ ✓ shape (4)

NOV 2019

QUESTION/*VRAAG* 9

9.1	$f'(x) = 9x^2$ $3x^3 = 9x^2$ $3x^3 - 9x^2 = 0$ $3x^2(x-3) = 0$ $x = 0$ or $x = 3$	✓ $f'(x) = 9x^2$ ✓ $x = 0$ ✓ $x = 3$ (3)
9.2.1	For f and f'	✓ answer (1)
9.2.2	The point (0 ; 0) is : A point of inflection of f A turning point of f'	✓ f: inflection point ✓ f' : turning point (2)
9.3	$f''(x) = 18x$ Distance $= f''(1) - f'(1)$ $= 18(1) - 9(1)^2$ $= 9$	✓ $f''(x) = 18x$ ✓substitution ✓answer (3)
9.4	$3x^3 - 9x^2 < 0$ $3x^2(x-3) < 0$ but $3x^2 > 0$  $\therefore x - 3 < 0$ $\therefore x < 3$, $x \neq 0$	✓ $3x^3 - 9x^2 < 0$ ✓ factors ✓ $x < 3$ ✓ $x \neq 0$ (4)
		[13]

NOV 2021

QUESTION/VRAAG 11

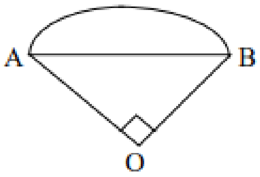
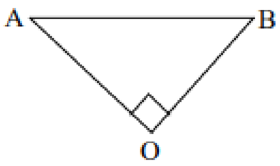
11	Time $= \frac{20}{x}$ Cost = (water cost per hour × time) + (kms × R/km) $C(x) = 1{,}6 \times \left(\frac{20}{x}\right) + 20\left(1{,}2 + \frac{x}{4000}\right)$ $C(x) = \frac{32}{x} + 24 + \frac{x}{200}$ $C'(x) = -\frac{32}{x^2} + \frac{1}{200} = 0$ $x^2 = 6400$ $x = 80\ km/h$	✓ $\frac{20}{x}$ ✓ $1{,}6 \times \left(\frac{20}{x}\right)$ ✓ $20\left(1{,}2 + \frac{x}{4000}\right)$ ✓ $C(x) = \frac{32}{x} + 24 + \frac{x}{200}$ ✓ $C'(x) = -\frac{32}{x^2} + \frac{1}{200}$ ✓ $C'(x) = 0$ ✓ answer (A) (7)
		[7]

NOV 2020

QUESTION/*VRAAG* 9

9.1	Total surface area $= 2\ell w + 2wh + 2\ell h$ but: $\ell = 3w$ Total surface area $= 6w^2 + 2wh + 6wh$ $C = 15(6w^2) + 6(2wh + 6wh)$ $= 15(6w^2) + 6(8wh)$ $= 90w^2 + 48wh$	✓ $2\ell w + 2wh + 2\ell h$ ✓ $\ell = 3w$ ✓ $15(6w^2)$ ✓ $6(2wh + 6wh)$ (4)
9.2	$5 = 3w^2h$ $h = \frac{5}{3w^2}$ $C = 90w^2 + 48wh$ $C(w) = 90w^2 + 48w\left(\frac{5}{3w^2}\right)$ $= 90w^2 + 80w^{-1}$ $C'(w) = 180w - 80w^{-2}$ $180w - 80w^{-2} = 0$ $180w^3 - 80 = 0$ $w^3 = \frac{80}{180}$ $w = \sqrt[3]{\frac{80}{180}}$ $w = 0{,}76$	✓ $h = \frac{5}{3w^2}$ ✓ substitution ✓ $C(w) = 90w^2 + 80w^{-1}$ ✓ derivative ✓ equating derivative to zero ✓ value of w (6)
		[10]

JUNE 2021

10.2.1		
	Area of segment $= \frac{1}{4}$ Area of big circle	
	$=\frac{1}{4}\pi\ (x-x^2)^2$	✓✓ $\frac{1}{4}\pi(x-x^2)^2$
		
	Area triangle ABO counted	
	$=$ Area $\Delta = \frac{1}{2}\left(x-x^2\right)^2$	✓ Area $\Delta = \frac{1}{2}\left(x-x^2\right)^2$
	Area of shaded region	✓ subtract areas
	$=\frac{1}{4}\pi(x-x^2)^2-\frac{1}{2}\left(x-x^2\right)^2$	
	$=\frac{\pi-2}{4}\left(x-x^2\right)^2$	✓ common factor
	$=\left(\frac{\pi-2}{4}\right)\left(x^2-2x^3+x^4\right)$	(5)

10.2.2	Area of shaded region	
	$=\frac{(\pi-2)}{4}\left(x^4-2x^3+x^2\right)$	
	$\frac{dA}{dx}=\left(\frac{\pi-2}{4}\right)\left(4x^3-6x^2+2x\right)$	✓ $\left(\frac{\pi-2}{4}\right)\left(4x^3-6x^2+2x\right)$
	$4x^3-6x^2+2x=0$	
	$x(2x^2-3x+1)=0$	
	$x(2x-1)(x-1)=0$	✓ factors
	$x \neq 0$ or $x=\frac{1}{2}$ or $x \neq 1$	✓ $x=0; x=1; x=\frac{1}{2}$
		✓ $x=\frac{1}{2}$ (4)
		[13]